Análise de Confiabilidade de um Pórtico Sujeito à Análise Limite sob Incerteza

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Resumo: Diante da variabilidade dos dados, torna-se relevante quantificar a confiabilidade de um sistema. Portanto, este artigo visa realizar uma análise de confiabilidade em um pórtico em relação ao seu estado limite plástico quando a carga aplicada e o limite de escoamento do material são incertos (parâmetros probabilísticos). O método de propagação de incertezas empregado é a simulação de Monte Carlo. Em vista dos resultados, a abordagem determinística não acusou falha para o pórtico, enquanto que a não determinística apontou probabilidade de falha de 0.49% para a estrutura. Portanto, para estruturas de baixa margem de segurança, a análise determinística traz o risco de uma falha estrutural não prevista.

Palavras-chave: Análise de confiabilidade; Análise limite; Incerteza; Pórtico.

Reliability Analysis of a Frame Subjected to Limit Analysis under Uncertainty

Abstract: Given the data variability, it becomes relevant to quantify the reliability of a system. Therefore, this paper aims to perform a reliability analysis of a frame in relation to its plastic limit state when the applied load and material yield strength are uncertain (probabilistic parameters). The method of propagation of uncertainties used is the Monte Carlo simulation. In view of the results, the deterministic approach did not show a failure for the frame, while the non-deterministic approach showed a failure probability of 0.49% for the structure. Therefore, for structures with a low safety margin, the deterministic analysis carries the risk of an unpredicted structural failure.

Keywords: Reliability analysis; Limit analysis; Uncertainty; Frame.

Introduction

Limit analysis (LA) associated with reliability analysis (RA) is within the research scope of some authors as these subjects are treated in the solution of the most diverse types of problems. A RA was performed over a LA in settled masonry arches, where analytical predictions were compared to numerical analyses and to experimental results [1]. Two trusses and one frame were subjected to a RA under stochastic uncertainty [2]. The probabilistic LA was used to assess the conditional probability of collapse of beam, truss, and frame [3]. A truss, a beam, and a column were subjected to a RA in what concerns collapse and allowable displacements [4]. A RA of spatial variance frames was performed based on random field and stochastic elastic modulus reduction method [5]. Related to LA, the upper bound theorem aims at finding values of load-carrying capacity greater than or equal to the true value of collapse factor α_R [6]. In terms of the upper collapse factor α_U, the internal work W can be compared to the external one $\sum P_i\, u_i$, where P_i is the i-th externally applied load, u_i refers to its corresponding displacement, F_i is the i-th internal force, and n is the number of internal forces. This aims at finding load values greater than or equal to the real collapse load. Thus, the collapse factor for a structural system can be calculated from Equation 1. The difference between α_U and α_R represents the surplus of the value calculated by the upper bound theorem and the real one:

$$\alpha_U = \frac{\int_V W\left(F_1, F_2, \Lambda\ , F_i, \Lambda\ , F_n\right) dV}{\sum P_i\, u_i} \geq \alpha_R\,. \tag{1}$$

In what concerns RA, first-order reliability method (FORM) is an integration method of analytical probability such that limit state function is estimated by a linear surface at the design point in the design space and it is employed to obtain the failure probabilities of loaded bodies [7]. Let R and S be the normally distributed limit and applied loads, respectively. If these variables are statistically independent, the variable $Z(\mathbf{X}) = R(\mathbf{X}) - S(\mathbf{X})$, a limit state function, will have a normal probability distribution too. Therefore, the failure corresponds to $R < S$ or $Z < 0$, where $\mathbf{X}$ is the vector related to the values of all possible realizations.

For normal probability distributions, the failure probability is dependent on the relation between the means μ_Z, μ_R, and μ_S, which are, respectively, related to limit state function, limit load, and applied load. Equivalently, the standard deviations λ_Z, λ_R, and λ_S are, respectively, the standard deviations for Z, R, and S. This relation is called reliability index β (Equation 2), restricted to normally distributed variables:

$$\beta = \frac{\mu_Z}{\lambda_Z} = \frac{\mu_R - \mu_S}{\sqrt{\lambda_R^2 + \lambda_S^2}}\,. \tag{2}$$

Thenceforth, the failure probability is given by Equation 3, where $z = (x - \mu)/\lambda$ is the standard normal variable, in which x is the normal variable, μ is the mean related to the normal variable, and λ is the standard deviation of the normal variable. The failure probability refers to the quantity of failures in the universe of realizations.

$$P_F = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-\beta} e^{-0.5\, z^2}\, dz\,. \tag{3}$$

Objective

The main objective of this work is to calculate the collapse factor, limit load, reliability index, and failure probability related to plastic collapse when uncertainties are modeled by both random applied loads and material yield strength.

Material and Methods

The frame calculated herein use elastic-perfectly plastic material, i.e., the strain hardening is not present when the material reaches and goes through its plastic zone. The referred frame is composed of three bars of identical cross-sections and is subjected to a mean and standard deviation of applied load of, respectively, $F^{M} = 120\,kN$, and $F^{SD} = 12\,kN$ at point B, which distance from the left pin support is $L = 1500\,mm$, as shown in Figure 1, part (a). The possible collapse mechanism considered on the analysis of the frame is presented in Figure 1, part (b), where θ is the angle formed between deformed and undeformed configurations of bar AB. The cross-section of the three structural elements is rectangular, with base $b = 50\,mm$ and height $h = 100\,mm$. The mean and standard deviation of the material yield strength are, respectively, $S_Y{}^{M} = 500\,MPa$ and $S_Y{}^{SD} = 50\,MPa$. The example was subjected to 10000 runs, observing the normal probability distribution.

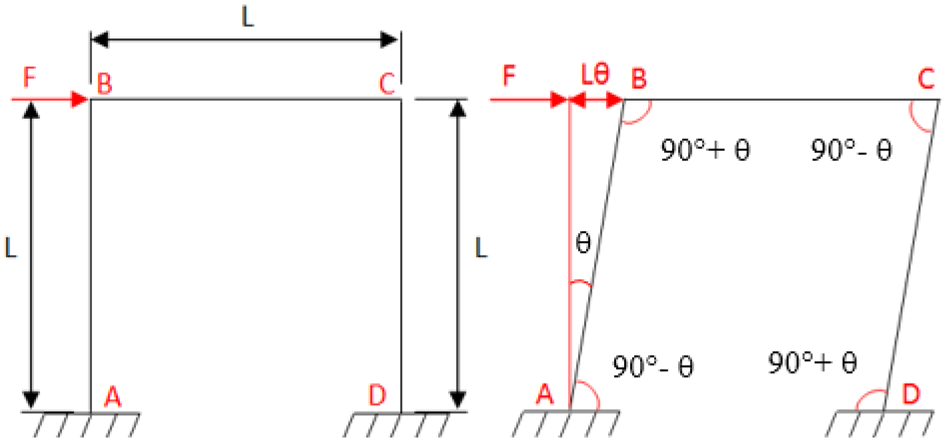

Figure 1. (a) Schematic model for a frame under application of load. (b) Collapse mechanism. Source: own authorship (2021).

Results

When the upper bound theorem is applied to obtain the limit load F_{LIM} , Equation 4 gives the following result:

$$F_{LIM} = 4\frac{M_P}{L} = 166666.67\ N\ , \tag{4}$$

where the plastic moment for a rectangular cross-section beam is given by $M_P = bh^2 S_Y / 4$. Thenceforth, Equation 5 gives the upper bound amplification factor, Equation 6 shows the calculated reliability index, and Equation 7, the failure probability:

$$\alpha_U = \frac{F_{LIM}}{F} = \frac{bh^2 S_Y}{FL} = 1.39\,. \tag{5}$$

$$\beta = 2.61, \tag{6}$$

$$P_F = 0.49\%, \tag{7}$$

Discussion

If the deterministic approach is considered, the system is 100% safe because the mean applied load is lower than the mean limit load. However, if the probabilistic approach is observed, the system is approximately 99.51% safe. It means that the frame under the described circumstances has approximately 0.49% of failure probability. The probability distribution functions are plotted in Figure 2.

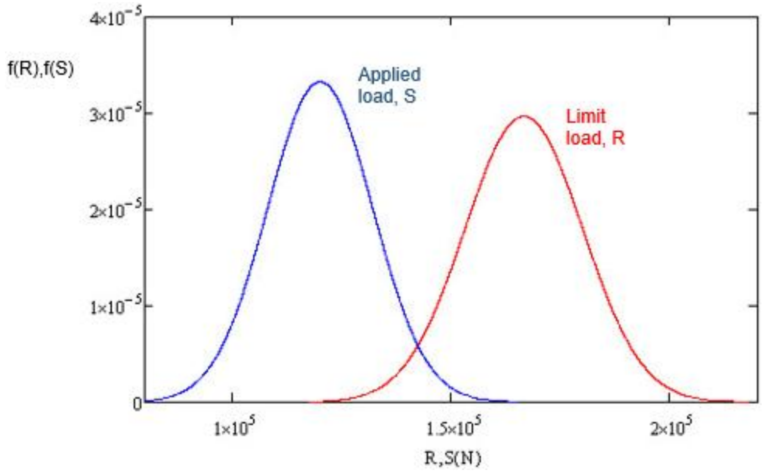

Figure 2. Distribution functions $f(S)$ and $f(R)$ for the frame case. Source: own authorship (2021).

If an elastic limit analysis is performed, the structure can support a load lower than that obtained by carrying out a limit analysis because the former approach focuses on avoiding the plasticization of a point or a line. The latter approach is based on avoiding the plasticization of enough entire cross-sections in order not to produce the sufficient number of mechanisms to provoke collapse. The other side of this trade-off is that, under the consideration of such a higher limit, the structure deforms and displaces much more than in the case of elastic design. In view of this, limit analysis takes advantage over the system capacity as a whole. Therefore, one of the manners to maximize the load carrying capacity of a structural system is to conduct a plastic limit analysis on this structure. Thus, the application of limit analysis turns the structure lighter. In designs where the safety margin is low, a deterministic analysis is associated to a more probable risk of a non-predicted system failure.

Additionally, if the objective of the structural analysis is to model it the more realistic as possible, the uncertainties should be considered, since they are inherent to common applications. Thus, if the uncertainties are assumed as part of the analysis process, it is relevant to consider the variables as being non-deterministic and calculate their probability of failure, which refers to the system reliability. Therefore, when considering the uncertainties, the structural system can be more effectively evaluated, which provides valuable information for the decision making process.

Conclusions

The general objective of calculating the collapse factor, limit load, reliability index, and failure probability has been achieved. As verified, for low values of safety margin, the structural system is more susceptible to unpredictable failures.

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