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Remarks on Paraconsistent Annotated Evidential Logic *E*τ

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Abstract

In this paper we make some observations and corrections concerning the lattice underlying the Paraconsistent Annotated Evidential Logic *E*τ.

Keywords: Paraconsistent logic, Paraconsistent annotated logics, lattice.

Considerações Sobre Lógica Paraconsistente Anotada Evidencial *E*τ

Resumo: Neste trabalho fazemos algumas observações e correções relativas a estrutura subjacente que fundamenta a lógica Paraconsistente Anotada Evidencial *E*τ.

Palavras chave: logica paraconsistente, lógica paraconsistente anotada, reticulado.

1. Introduction

The aim of this paper is to make some corrections and observations concerning to a special case of so called Annotaded logics. They are a new kind of two-sorted logics, in general a non-alethic logics (paraconsistent and paracomplete) [ABE, 2006].

In what follows, we summarize some basic definitions and conventions discussed in the paper. Let *T* be a theory whose underlying logic is *L*. *T* is called inconsistent when it contains theorems of the form A and ¬A (the negation of A). If *T* is not inconsistent, it is called consistent. *T* is said to be trivial if all formulas of the language of *T* are also theorems of *T*. Otherwise, *T* is called non-trivial.

When L is classical logic (or one of several others, such as intuitionistic logic), T is inconsistent iff T is trivial. So, in trivial theories the extensions of the concepts of formula and theorem coincide.

A paraconsistent logic is a logic that can be used as the basis for inconsistent but non-trivial theories. A theory is called paraconsistent if its underlying logic is a paraconsistent logic.

Issues such as those described above have been appreciated by many logicians. In 1910, the Russian logician Nikolaj A. Vasil'év (1880-1940) and the Polish logician Jan Lukasiewicz (1878-1956) independently glimpsed the possibility of developing such logics.

Nevertheless, Stanislaw Jaskowski (1996-1965) was in 1948 effectively the first logician to develop a paraconsistent system, at the propositional level. His system is known as 'discussive' (or discursive) propositional calculus'. Independently, some years later, the Brazilian logician Newton C.A. da Costa (1929-) constructed for the first time hierarchies of paraconsistent propositional calculi C_i, $1 \leq i \leq \omega$ of paraconsistent first-order predicate calculi (with and without equality), of paraconsistent description calculi, and paraconsistent higher-order logics (systems NF_i, $1 \leq i \leq \omega$).

Another important class of non-classical logics are the paracomplete logics. A logical system is called paracomplete if it can function as the underlying logic of theories in which there are formulas such that these formulas and their negations are simultaneously false. Intuitionistic logic and several systems of many-valued logics are paracomplete in this sense (and the dual of intuitionistic logic, Brouwerian logic, is therefore paraconsistent).

As a consequence, paraconsistent theories do not satisfy the principle of non-contradiction, which can be stated as follows: of two contradictory propositions, i.e., one of which is the negation of the other, one must be false. And, paracomplete theories do not satisfy the principle of the excluded middle, formulated in the following form: of two contradictory propositions, one must be true.

Finally, logics which are simultaneously paraconsistent and paracomplete are called non-alethic logics.

Problems of various kinds give rise to these non-classical logics: for instance, the paradoxes of set theory, the semantic antinomies, some issues originating in dialectics, in Meinong's theory of objects, in the theory of fuzziness, and in the theory of constructivity. However, one of the most amazing applications was obtained in recent years in Artificial Intelligence, Automation and Robotics, Engineering, Psychoanalysis, Philosophy, among other fields.

Throughout this paper, usual conventions and notions of naive set theory are assumed without extensive comments.

2. Paraconsistent Annotated Evidential Logic Eτ

The atomic formulas of the language of the logic $E\tau$ are of the type $p_{(\mu, \lambda)}$, where $(\mu, \lambda) \in [0, 1]^2$ and [0, 1] is the real unitary interval (p denotes a propositional variable).

$p_{(\mu, \lambda)}$ can be intuitively read: "It is assumed that p's favorable evidence is μ and contrary evidence is λ." Depending on the applications, other terms can be considered instead 'evidence', such as 'probability', 'belief', etc.

If we consider the proposition p = "The way is obstacle free" and suppose that we have the following situations: "The way is obstacle free" (in the Logic Eτ we can express this fact as $p_{(1, 0)}$) "The way is not obstacle free" (in the Logic Eτ we can express this fact as $p_{(0, 1)}$)

Such inconsistent situation can be expressed in the Logic Eτ as $p_{(1, 1)}$ (which means intuitively that the robot's way information is in a conflicting state).
Thus:

- $p_{(1.0, 0.0)}$ can be read as a true proposition.
- $p_{(0.0, 1.0)}$ can be read as a false proposition.
- $p_{(1.0, 1.0)}$ can be read as an inconsistent proposition.
- $p_{(0.0, 0.0)}$ can be read as a paracomplete (unknown) proposition.
- $p_{(0.5, 0.5)}$ can be read as an indefinite proposition.

We consider logical connectives $\neg$ ((weak) negation), $\wedge$ conjunction), $\vee$ (disjunction), $\rightarrow$ (implication), $\leftrightarrow$ (equivalence), and so on. Complex formulas of the logic Eτ are obtained by combinations of the logical connectives, as usual.

One important observation is regarding the negation operator: it is intuitive that $\neg p_{(1.0, 1.0)} \leftrightarrow p_{(0.0, 1.0)}$. Also $\neg p_{(0.1, 1.0)}$. when we have other non-extreme values for annotation constants, we can get, for instance, the following equivalence: $\neg p_{(0.8, 0.4)} \leftrightarrow p_{(0.4, 0.8)}$ (among other possible equivalences).

We have adopted the this property for negation. Technically, this means the following: there is an operator on τ. $\sim: |\tau| \rightarrow |\tau|$, $\sim(\mu, \lambda) = (\lambda, \mu)$.

Observe that in this definition, the negation of a "true" proposition is equivalent to a "false" proposition, the negation of a "false" proposition is equivalent to a "true" proposition, the negation of an "inconsistent" proposition is equivalent to an "inconsistent" proposition, and the negation of a "paracomplete" proposition is equivalent to a "paracomplete" proposition.

Thus we symbolize in this way:

- $T \equiv$ Inconsistent state
- V ≡ True state
- F ≡ False state
- $\perp \equiv$ Paracomplete state

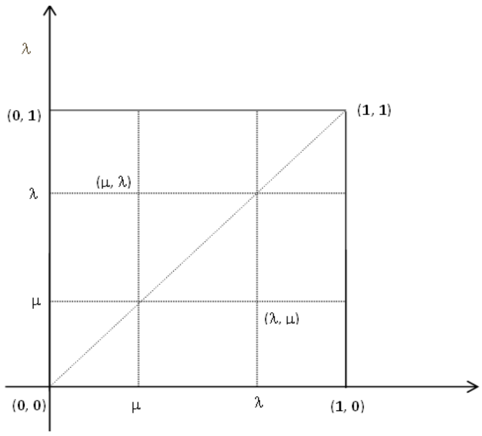

Figure 1 - Negation operator.

In the set [0, 1]×[0, 1] we define a relation by (μ1, λ1) ≤ (μ2, λ2) ⇔ μ1 ≤ μ2 e λ2 ≤ λ1. Such system constitutes a lattice. We have

An order relation is defined on $[0, 1]^2$: (μ1, λ1) ≤ (μ2, λ2) ⇔ μ1 ≤ μ2 and λ2 ≤ λ1, constituting a lattice that will be symbolized by τ.

Properties:
1. ∀μ, λ ∈ τ, (μ, λ) ≤ (μ, λ) (reflexivity)
2. ∀μ1, λ1, μ2, λ2 ∈ τ, (μ1, λ1) ≤ (μ2, λ2) e (μ2, λ2) ≤ (μ1, λ1), entail (μ1, λ1) = (μ2, λ2) (anti-symmetry)
3. ∀μ1, λ1, μ2, λ2, μ3, λ3∈ τ, (μ1, λ1) ≤ (μ2, λ2) e (μ2, λ2) ≤ (μ3, λ3), entail (μ1, λ1) ≤ (μ3, λ3) (transitividade)
4. ∀μ1, λ1, μ2, λ2 ∈ τ, there exists the supremum of {(μ1, λ1), (μ2, λ2)} symbolized by (μ1, λ1) ∨ (μ2, λ2) = (Max{μ1, λ2}, Min{λ1, λ2})

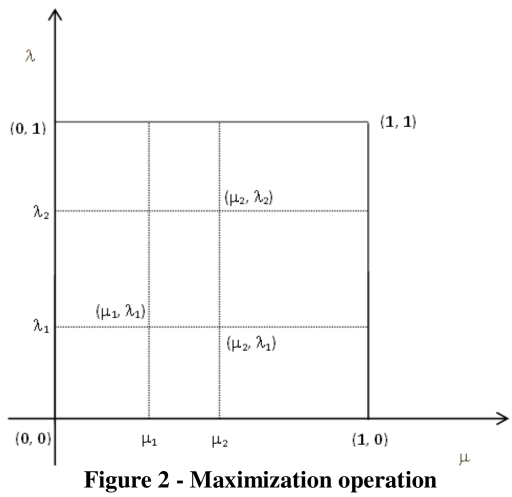

Figure 2 - Maximization operation

5. ∀μ1, λ1, μ2, λ2 ∈ τ, there exists the infimum of {(μ1, λ1), (μ2, λ2)} symbolized by (μ1, λ1) ∧ (μ2, λ2) = (Min{μ1, λ2}, Max{λ1, λ2})
6. ∀μ, λ ∈ τ, (0, 1) ≤ (μ, λ) ≤ (1, 0)
7. ∀μ1, λ1, μ2, λ2 ∈ τ, there is the supremum of {(μ1, λ1), (μ2, λ2)} indicated by (μ1, λ1) ∨ (μ2, λ2) = (Max{μ1, λ2}, Min{λ1, λ2})
8. ∀μ1, λ1, μ2, λ2 ∈ τ, there exists the infimum of {(μ1, λ1), (μ2, λ2)} indicated by (μ1, λ1) ∧ (μ2, λ2) = (Min{μ1, λ2}, Max{λ1, λ2})

The operator ∨ has the following properties:
1) For any (μ1, λ1) and (μ2, λ2) ∈ τ, we have (μ1, λ1) ∨ (μ2, λ2) = (μ2, λ2) ∨ (μ1, λ1) (Comutativity)
2) For any (μ1, λ1), (μ2, λ2) and (μ3, λ3) ∈ τ, we have [(μ1, λ1) ∨ (μ2, λ2)] ∨ (μ3, λ3) = (μ1, λ1) ∨ [(μ2, λ2) ∨ (μ3, λ3)] (Associativity)
3) For any (μ, λ) ∈ τ, we have (μ, λ) ∨ (μ, λ) = (μ, λ) (Idempotency)
4) For any (μ, λ) ∈ τ, we have (μ, λ) ∨ (0, 1) = (μ, λ) (Neutral element)
5) For any (μ, λ) ∈ τ, we have (μ, λ) ∨ ~(μ, λ) = (Max{μ, λ}, Min{μ, λ))

The operator ∧ has the following properties:
1) For any (μ1, λ1) and (μ2, λ2) ∈ τ, we have (μ1, λ1) ∧ (μ2, λ2) = (μ2, λ2) ∧ (μ1, λ1) (Comutativity)
2) For any (μ1, λ1), (μ2, λ2) and (μ3, λ3) ∈ τ, we have [(μ1, λ1) ∧ (μ2, λ2)] ∧ (μ3, λ3) = (μ1, λ1) ∧ [(μ2, λ2) ∧ (μ3, λ3)] (Associativity)
3) For any (μ, λ) ∈ τ, we have (μ, λ) ∧ (μ, λ) = (μ, λ) (Idempotency)
4) For any (μ, λ) ∈ τ, we have (μ, λ) ∧ (1, 0) = (μ, λ) (Neutral element)
5) For any (μ, λ) ∈ τ, we have (μ, λ) ∧ ~(μ, λ) = (Min{μ, λ}, Max{μ, λ))

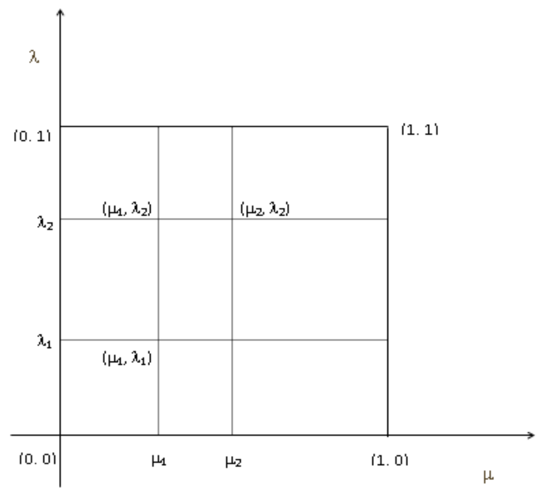

Figure 3 - Minimization operation

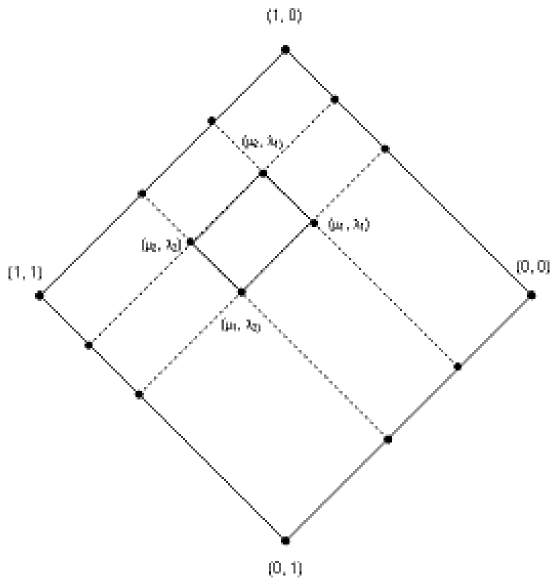

Figure 4 - Lattice τ

We introduce the following concepts (all considerations are taken with $0\le \mu, \lambda\le 1$):
Segment perfectly defined AB: $\mu + \lambda - 1 = 0$; $0 \le \mu, \lambda \le 1$
Segment perfectly undefined DC: $\mu - \lambda = 0$; $0 \le \mu, \lambda \le 1$

- Uncertainty degree: $Gun(\mu, \lambda) = \mu + \lambda - 1$ (1)
- Certainty degree: $Gce(\mu, \lambda) = \mu - \lambda$ (2)

We Introduce the following applications Gic:$[0, 1]\times[0, 1] \to [0, 1]$, Gpa:$[0, 1]\times[0, 1] \to [-1, 0]$, Gve:$[0, 1]\times[0, 1] \to [0, 1]$, Gfa:$[0, 1]\times[0, 1] \to [-1, 0]$ defined by:

- Inconsistency degree: $Gic(\mu, \lambda) = \mu + \lambda - 1$, provided that $\mu + \lambda - 1 \ge 0$
- Paracompleteness degree: $Gpa(\mu, \lambda) = \mu + \lambda - 1$, provided that $\mu + \lambda - 1 \le 0$
- Truth degree: $Gve(\mu, \lambda) = \mu - \lambda$, provided that $\mu - \lambda \ge 0$
- Falsity degree: $Gfa(\mu, \lambda) = \mu - \lambda$, provided that $\mu - \lambda \le 0$

With the uncertainty and certainty degrees we can get the following 12 output states (Table 1): *extreme states*, and *non-extreme states*. We observe that the present lattice is only to present the theory. In real applications, it must be adapted, naturally.

Extreme states	Symbol
True	V
False	F
Inconsistent	T
Paracomplete	⊥

Non-extreme states	Symbol
Quasi-true tending to Inconsistent	QV→T
Quasi-true tending to Paracomplete	QV→⊥
Quasi-false tending to Inconsistent	QF→T
Quasi-false tending to Paracomplete	Qf→⊥
Quasi-inconsistent tending to True	QT→V
Quasi-inconsistent tending to False	QT→F
Quasi-paracomplete tending to True	Q⊥→V
Quasi-paracomplete tending to False	Q⊥→F

Table 1. Extreme and Non-extreme states

Some additional control values are:

- V_{cin} = maximum value of uncertainty control
- V_{cve} = maximum value of certainty control
- V_{cpa} = minimum value of uncertainty control
- V_{cfa} = minimum value of certainty control

All states are represented in the next figure (Fig. 5).

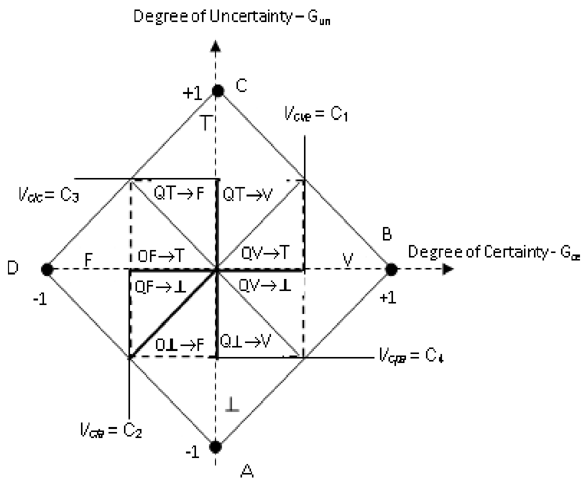

Figure 5 - Extreme and Non-extreme states.

3. Usefulness of the logic Eτ

The logic Eτ is one of the most useful logic among the so called paraconsistent logics. Its applicability in Automation and Robotics was pointed out v.g. in [DA SILVA FILHO *et al.,* 2006], [TORRES, 2009]. Electronic implementation of many logical controllers is in, v.g., [DA SILVA FILHO, 1999], and a ANN theory also was proposed in, v.g., [DA SILVA FILHO *et al.,* 2010]. Some applications of ANN based on logic Eτ are in [LOPES, 2009], [AMARAL,2013], [SOUZA, 2013]. Aspects on mathematical foundations were studied in [ABE, 2006].

4. Discussion

In this work we presented some corrections concerning the underlying lattice to the logic *E*τ. Some basic properties are also discussed. Although in many previous works employing the logic Eτ the correction mentioned here are not so crucial in their consequences; however the amendment is really necessary.

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